

NUMERICAL INVESTIGATION OF FLOW IN FIBROUS POROUS MEDIA

A. Tamayol* and M. Bahrami^o

*PhD Candidate, ata42@sfu.ca, ^oAssistant Professor, mbahrami@sfu.ca
Mechatronic Systems Engineering, School of Engineering Science,
Simon Fraser University, BC, Canada

ABSTRACT

Fibrous porous materials involve in many natural and engineering processes. In this study, the permeability of ordered fibrous media towards normal and parallel flows is studied numerically. Porous material is represented by a “unit cell” which is assumed to be repeated throughout the media and 1D fibers are modeled as a combination of Channel-like conduits. The numerical results are successfully compared with the experimental data collected by others and existing correlations in the pertinent literature, for square arrangement of fibers over a wide range of porosity. A compact correlation is proposed for parallel permeability of fibrous media, which can predict the results accurately over the entire range of porosity. Moreover, microscopic velocity profiles in the unit cell are investigated in detail for both parallel and normal flows. The numerically predicted velocity distributions for normal and parallel flow cases are compared with experimental and simple parabolic profiles. The comparison showed that although assumption of the parabolic velocity distribution results in reasonable values for permeability, this form of the velocity profile is not realistic.

NOMENCLATURE

d	Fiber diameter, m
K	Viscous permeability, m^2
K^*	Non-dimensional permeability, $K^* = K/d^2$
L	Unit cell length in Eq. (3), m
P	Pressure, Pa
Q	Volumetric flow rate, m^3/s
S	Distance between adjacent fibers in square arrangement, m
u	Velocity in x -direction, m/s
U_D	Volume-averaged superficial velocity, m/s
w	Velocity in z -direction, m/s

Greek symbols

δ	Distance between adjacent fibers in square arrangement in y -direction, m
ε	Porosity
μ	Fluid viscosity, $N \cdot s/m^2$
φ	Solid volume fraction, $\varphi = 1 - \varepsilon$

INTRODUCTION

Fibrous porous materials have applications in several engineering areas including: filtration and separation of particles, physiological systems, composite fabrication, heat exchangers, thermal insulations, and fuel cells [1-3]. Transport phenomena in porous media have been the focus of numerous studies since 19th century, which indicates the importance of this topic. Prediction of the velocity-field plays a key role in estimating the properties of porous media e.g. permeability. This can be achieved by using Darcy’s equation which assumes a linear relationship between the volume-averaged superficial fluid velocity, U_D , and the pressure gradient:

$$-\nabla P = \frac{\mu}{K} U_D \quad (1)$$

where, μ is the fluid viscosity and K is the permeability of the

medium. Darcy’s relationship is empirical, convenient, and widely accepted. However, Darcy’s equation holds when flow is in creeping regime [4]. To use Darcy’s equation we need to know the permeability of the medium beforehand. Permeability, a measure of the flow conductance of the solid matrix, depends on several factors including: porosity, particles shape and size distribution and particles arrangement. The permeability is calculated either experimentally or through microscopic analysis of porous media.

Studies on the permeability of fibrous media dates back to experimental works of Carman [5] and Sullivan [6] in 1940s and theoretical analyses of Kuwabara [7], Hasimoto [8], Happel and Brenner [9], and Sparrow and Loeffler [10] in 1950s. Kuwabara [7], solving vorticity transport and stream function equations and employing limited boundary layer approach, predicted the permeability of flow normal to randomly arranged fibers for materials with high porosities. Hasimoto [8] and Sparrow and Loeffler [10] used series solutions for determining the permeability of ordered arrangement of cylinders to normal and parallel flow, respectively. Happel and Brenner [9] analytically solved the Stokes equation for parallel and normal flow to a single cylinder with free surface model (limited boundary layer). The boundary conditions used by Happel and Brenner [9] were different from the Kuwabara’s study [7]. They [9] hypothesized that the flow resistance of a random 3D fibrous structure is equal to one third of the parallel plus two third of the normal flow resistances of 1D array of cylinders [4]. Later, Sangani and Acrivos [11], performed analytical and numerical studies on viscous permeability of square and staggered arrays of cylinders for the entire range of porosity, when their axes were perpendicular to the flow direction. Their analytical models were accurate for the lower and higher limits of porosity [11]. Sangani and Yao [12] extended the studies of [11] to random media and reported numerical results for the permeability of random 1D fibers towards

normal and parallel flows. Drummond and Tahir [13] solved Stokes equations for normal and parallel flow towards different ordered structures. They used a distributed singularities method to find the flow-field in square, triangular, hexagonal and rectangular arrays. They [13] compared their results with numerical values of Sangani and Acrivos [11] for normal flow and Happel and Brenner [9] for the parallel case. The model of Drummond and Tahir [13] for normal permeability was very close to the analytical model of Sangani and Acrivos [11], thus, it is only accurate for highly porous materials [13]. Sahraoui and Kaviani [14] included inertial effects and numerically determined the permeability of cylinders to normal flow. They also proposed a correlation which was accurate in limited range of porosity, i.e., $0.4 < \varepsilon < 0.7$. Using numerical simulations, Van der Westhuizen and Du Plessis [15] proposed a correlation for the normal permeability of 1D fibers. Theoretical prediction of the permeability of general triangular arrangement was presented by Hellou et al. [16]. They also proposed a correlation for determination of permeability of periodic triangular arrangements. Recently, Sobera and Kleijn [17] studied the permeability of random 1D and 2D fibrous media both analytically and numerically. Their analytical model was a modification of the scale analysis proposed by Clauge et al. [1]. A comparison of the model of Sobera and Kleijn [17] with numerical results showed that their model was accurate in highly porous materials [17]. However, the difference in low porosities was significant. The reported relationship for permeability of random fibers was a function of fibers distance and a non-dimensional randomness number [17]. Several investigations were devoted to quantifying the permeability of random fibrous media, i.e., real materials. Several researchers related the permeability of random media to the values of parallel and normal permeability of 1D fibers. The model proposed by Jackson and James [18] was based on this approach. However, limitations of this model were shown in the literature [2]. Tomadakis and Sotirchos [19] proposed a model for anisotropic permeability through 1D, 2D, and 3D overlapping random fibrous beds. Although the model of [19] were meant to cover all types of random fibrous media, in some cases the errors between the model and experimental data were considerable; see Ref. [2] for more details. Using the similarity between permeability and electrical conduction in porous media, Avellaneda and Torquato [20] proposed an upper bound for the permeability of fibrous media. Later, Tomadakis and Robertson [2] showed that this bound was violated by several data points available in the literature. Using experimental data, Tomadakis and Robertson [2] stated that the upper and lower bounds for fibrous media with random orientation of fibers were normal and parallel permeability of 1D arrangements.

Several experimental studies have been conducted for determination of the permeability of fibrous media [22-28]. Comprehensive reviews of these experimental works are available in Jackson and James [18], Astrom et al. [29], and Tomadakis and Robertson [2].

Our literature review can be summarized:

- 1) Less attention has been paid to determination of parallel permeability of ordered fibrous materials
- 2) A majority of the existing correlations for permeability

are based on curve-fitting of experimental or numerical data, i.e., not based on analytical modelings

3) Most of the theoretical models found in the literature are not general and fail to predict the permeability over the entire range of porosity

4) In most of analytical investigations micro-scale velocity distribution has not been verified.

In most engineering applications prediction of the exact velocity profile is not necessary and knowing the macroscopic properties such as permeability is sufficient; however, microscopic flow-field is needed for determining important phenomena such as particle deposition and mass transfer rates.

Recently, Tamayol and Bahrami [21] studied permeability of touching and non-touching ordered fibrous media towards normal and parallel flow. Analytical models were developed using the concept of “unit cell” and introducing an “integral technique”. Assuming a parabolic velocity profile within the unit cells and integrating the continuity and momentum equations, compact analytical relationships were reported for pressure drop and permeability of considered patterns [21]. They [21] also showed that the proposed normal flow permeability of square unit cell predicts the trends observed in experimental data and serves as a lower bound for the permeability of fibrous media. The proposed model was successfully validated against experimental data collected from several sources over a wide range of porosity, fibrous materials, and fluids. Due to lack of experimental data for parallel permeability, Tamayol and Bahrami [21] were not able to verify their model for this problem.

The objectives of the present study are:

1) Develop numerical simulations to study the details of velocity field in ordered fibrous media. This will provide an independent means to verify our previous study [21]

2) Use of the numerical results for verifying the parallel flow model where a lack of experimental data exists in the literature.

In this study the unit cell concept is used to determine the normal and parallel permeability of 1D fibrous media numerically; governing equations are solved using Fluent software [30]. Using the predicted values of pressure drop and relevant permeabilities, a compact correlation is also proposed for parallel permeability of 1D fibers which is easy-to-use and more importantly accurate over the entire range of porosity. Second part of the paper is devoted to comparison of the calculated velocity profiles with those assumed by Tamayol and Bahrami [21] and the experimental data reported by Zhong et al. [27].

MODELING APPROACH

Darcy’s relationship, Eq. (1), is valid when the flow passing through pores is in creeping regime, i.e., inertial effects are negligible. Therefore, the Stokes and continuity equations govern the flow-field. The flow is also assumed to be incompressible, steady state and porous media is completely saturated. We used Fluent software [30]; since this software solves complete Navier-Stokes (NS) equations, the flow Reynolds number should be kept sufficiently low to ensure negligible effects of inertial terms. Therefore, the inlet

velocity is set low enough to guarantee that the Reynolds number based on the fibers diameter, d , is below 0.05 for all cases; thus, creeping flow exists.

The 1D fibrous media are studied in the present work. Following the approach used successfully in several applications such as spherical packed beds [31], a unit cell is considered for analyzing the media. The unit cell (or basic cell) is the smallest volume which can represent the characteristics of the whole microstructure. Porous media are deemed to be periodic and the unit cells repeat throughout the material. Although the velocity distribution in porous material is 3D, because of the large length of fibers, variations in the z -direction can be neglected for the normal flow case, see Fig. 1. Figures 1 and 2 show how unit cells are produced for simulating normal and parallel flows. The unit cells are selected as the space between adjacent cylinders as shown in Figs. 1 and 2. For convenience and without loss of generality, the unit cells are assumed to be square. The same approach can be followed for rectangular unit cells. Porosity for this arrangement can be determined from:

$$\varepsilon = 1 - \frac{\pi d^2}{4S^2} \quad (2)$$

where d is the fibers diameter and S is the distance between adjacent fibers. Since analytical relationships available in the literature for permeability are reported for fully developed flow, the selected geometries have to be arranged so that the fully developed condition is reached. To achieve this, a set of 7-10 unit cells in series are considered (see Fig. 3) and velocity profiles are compared at the entrance to each unit cell. For parallel flows, the length of the cylinders is assumed to be $40d$. This length of the unit cell is expected to result in fully developed velocity profile.

A finite volume approach is used for solving the governing equations and second order upwind scheme is selected to discretize the governing equations. SIMPLE algorithm is employed for pressure-velocity coupling. The inlet velocity

of the media is assumed to be uniform. Creeping flow regime results in fast development of velocity profile. As a result, the outflow boundary condition is applied in the computational domain outlet, i.e., normal gradient of properties along the outlet is zero and the values of all properties at the outlet are interpolated from the computational domain. The symmetry boundary condition is applied on the side borders of the considered unit cells; this means that normal velocity and gradient of parallel component of the velocity on the side borders are zero.

RESULTS AND DISCUSSIONS

Numerical results are obtained from 2D and 3D analyses for normal and parallel flows, respectively. Different numerical grids are employed to check the grid independency of the results, which for brevity are not included here. In the following subsections our focus will be on the pressure drop/permeability and velocity distribution for the parallel and the normal flows.

Normal flow

Permeability can be calculated from Darcy's relationship:

$$K = \frac{\mu L}{\Delta P} U_D \quad (3)$$

where ΔP is the pressure drop in the unit cell, U_D is the volume averaged velocity, and L is the unit cell length. Once the pressure drop is known, permeability can be evaluated from Eq. (3). The pressure drops used for calculation of permeability are the values obtained from the developed regions. The inlet velocity is set equally for all cases; therefore, using continuity equation, U_D can be calculated from the following relationship:

$$U_{Dnormal} = u_{inlet} \frac{S-d}{S} \quad (4)$$

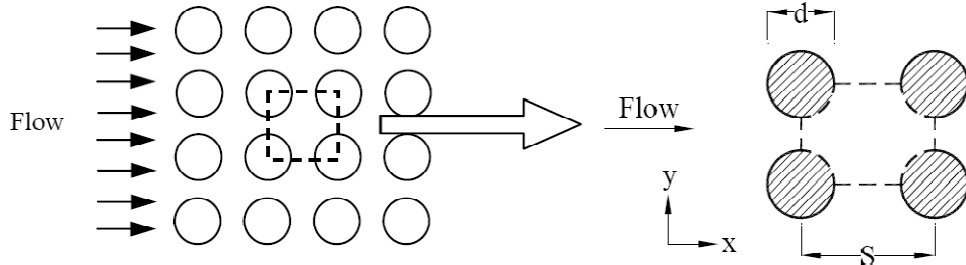


Figure 1: Unit cell for modeling normal flow.

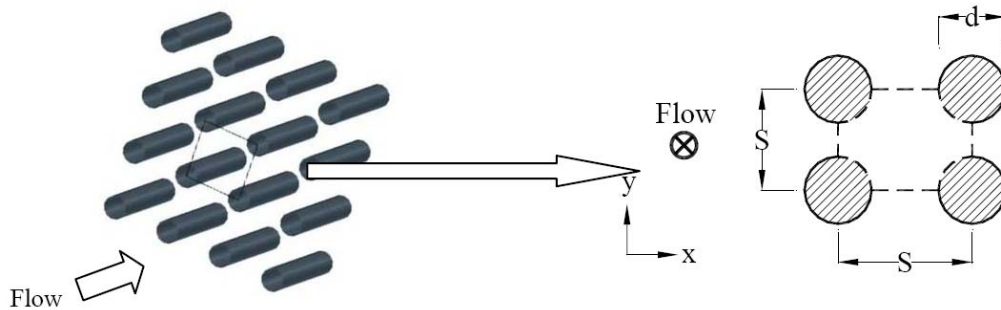


Figure 2: Unit cell for modeling parallel flow.

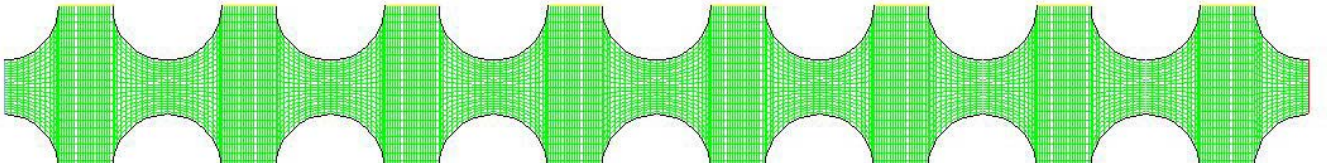


Figure 3: Typical numerical grid used for modeling normal flow, $\varepsilon = 0.65$.

For the normal flow, L is equal to S in Eq. (3). The predicted values for the non-dimensional permeability, $K^* = K/d^2$, and the related parameters for the normal flow cases are reported in Table 1. In Fig. 4 the numerical results of permeability are compared with experimental data collected from several sources. As can be seen, numerical results are in agreement with experimental data. These experiments were conducted using different fluids including: air, water, oil, and glycerol with a variety of porous materials such as metallic rods, glass wool, and carbon fibers. The results of the present numerical simulations are compared with the model reported in our previous study [21]. The comparison show that the analytical model of Tamayol and Bahrami [21] captures the trends of the numerical results over the entire range of the porosity.

Table 1: The numerical results of normal permeability and related parameters, $d = 1(\text{cm})$ and $u_{inlet} = 0.05 (\text{m/s})$

ε	$S(\text{cm})$	$U_D(\text{m/s})$	$\Delta P (\text{Pa})$	K^*
0.45	1.195	0.0081	61	0.0016
0.65	1.498	0.0166	17.1	0.0146
0.8	1.981	0.0248	6.67	0.0736
0.85	2.288	0.0281	4.9	0.1314
0.9	2.802	0.0322	2.9	0.3107
0.95	3.962	0.0374	1.6	0.9257
0.98	6.265	0.0420	0.77	3.4188
0.995	12.530	0.0460	0.27	21.352

A comparison of the numerical results and several existing analytical models is also presented in Fig. 6; note the following:

- All of the models could capture trends of data in higher limits of porosity, i.e., $\varepsilon > 0.9$
- Only the model proposed by Tamayol and Bahrami [21] predicts the trends of data accurately over the entire range of porosity.

Determination of the exact micro-scale velocity profile requires detail knowledge of the geometry of the medium which is not feasible for porous media in general. Although for most engineering applications we are not interested in the exact form of the flow-field and need to know the pressure drop over the basic cell to calculate the permeability, knowing the exact micro-scale velocity distribution helps one to understand the nature of many engineering processes such as separation of particles and filtration. Figure 5 proves that the model of Tamayol and Bahrami [21] predicts the numerical and experimental data; this model is based on integral technique solution and parabolic velocity profile

assumption:

$$u = \frac{1}{2\mu} \frac{dP}{dx} (\delta^2 - y^2), \quad 0 \leq x \leq \frac{d}{2} \quad (5)$$

$$u = \frac{1}{2\mu} \frac{dP}{dx} \left(\frac{S^2}{4} - y^2 \right) + g(\varepsilon) \frac{2x}{S-d} \bar{U}, \quad \frac{d}{2} \leq x \leq \frac{S}{2} \quad (6)$$

where δ is the distance between adjacent cylinders:

$$\delta = \frac{S}{2} - \sqrt{\frac{d^2}{4} - x^2} \quad (7)$$

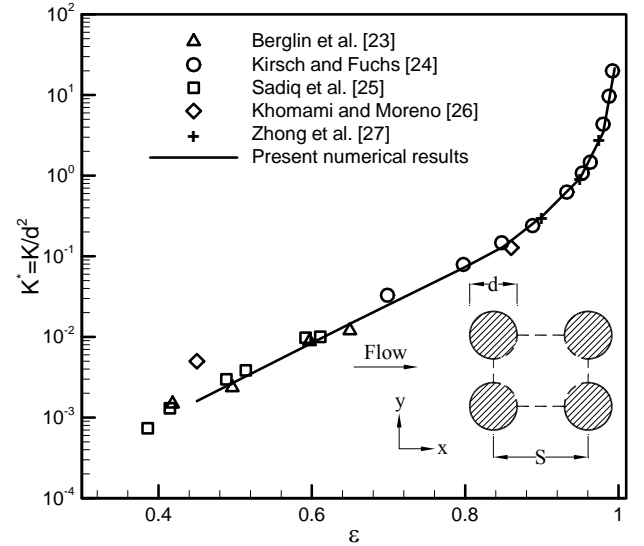


Figure 4: Comparison between the present numerical results and experimental data, normal flow.

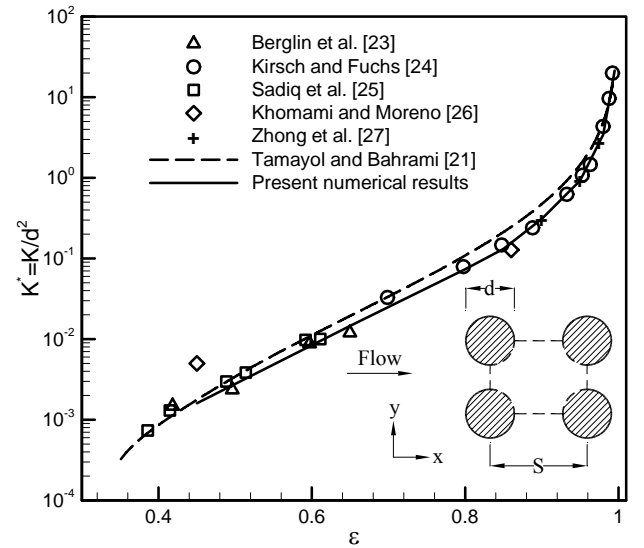


Figure 5: Comparison between the present numerical results, experimental data, and the model of [21], normal flow.

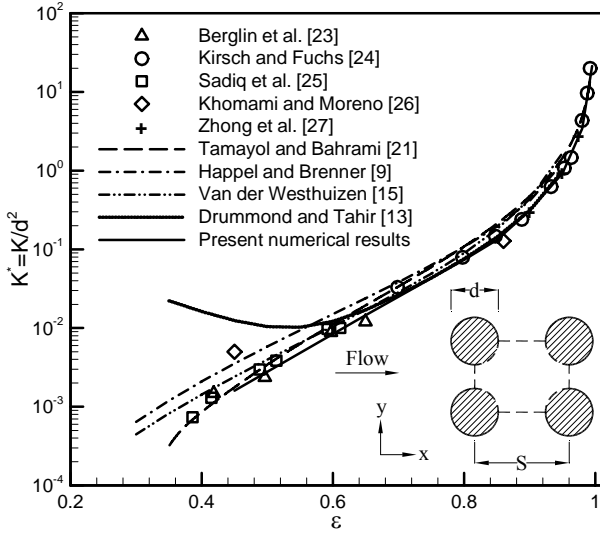


Figure 6: Comparison between the present numerical results, experimental data, and existing models, normal flow.

The last term in Eq. (6) was added to include the effects of the fluid velocity on the unit cell border lines (see Fig. 1) and $g(\epsilon)$ was assumed to be a linear function of porosity:

$$g(\epsilon) = 1.274\epsilon - 0.274 \quad (8)$$

In Fig. 7 the numerical velocity profiles are compared with the experimental data of Zhong et al. [27] for the normal flow through the fibrous media with $\epsilon = 0.9$ in several locations. Zhong et al. [27] measured the micro-scale velocity distribution in the porosity range of $0.9 < \epsilon < 0.975$. The solid structures were made of metallic rods and the working fluid was glycerol-water mixture. The Reynolds number based on the bulk velocity and the rod diameter, ranged from 0.01 to 0.17; a PIV technique was employed for measuring the flow velocity. As shown in Fig. 7 the present numerical results and the experimental profiles are in good agreement.

The numerical and parabolic velocity profiles [21] are plotted in several locations of the unit cell in Fig. 8 where $\epsilon = 0.9$. Figure 8 shows that the parabolic velocity assumption holds only near the entrance region of the unit cell. As the flow moves towards the center of the cell, the velocity profile changes significantly from the presumed parabolic distribution and the numerical and analytical velocity profiles do not agree. This can be a result of the abrupt change in the unit cell.

It should be noted that the permeability is not considerably affected by the assumption of unreal velocity distribution in the central region of the unit cell. This is due to the fact that most of pressure drop (directly related to the permeability) occurs at the entrance/exit regions of the unit cell where the parabolic velocity is in good agreement with the present numerical and the experimental data of Zhong et al. [27] and the defects are compensated with the unrealistic pressure drop in the central region of the unit cell.

Parallel flow

Parallel permeability is calculated from Eq. (3). The volume averaged and the inlet velocities for parallel flows are related:

$$U_{D_{parallel}} = u_{inlet} \epsilon \quad (9)$$

Using the values of the pressure drop obtained from the numerical simulations, parallel permeabilities are listed in Table 2.

Numerical values of the parallel permeability of the square arrangement of cylinders are plotted in Fig. 9. Only two experimental data points were found in the open literature. Therefore, for the verification of the numerical results these values are also compared to the numerical data reported by Sangani and Yao [12]. Figure 9 shows that the present simulation results are in agreement with the experimental and the numerical data.

In the Fig. 10 results of the numerical simulations are compared to the analytical models of Happel and Brenner [9], Sparrow and Loeffler [10], Drummond and Tahir [13], and Tamayol and Bahrami [21] as well as experimental data reported by Sullivan [6] and Skartsis and Kardos [28]; one can conclude:

- All of the models capture the trends of numerical data in the high limit of porosity, i.e., $\epsilon > 0.8$
- Only the models proposed by [13] and [21] are accurate in lower porosities, i.e., $\epsilon < 0.3$
- The model of Drummond and Tahir [13] predict the parallel permeability the over a wide range of porosity.

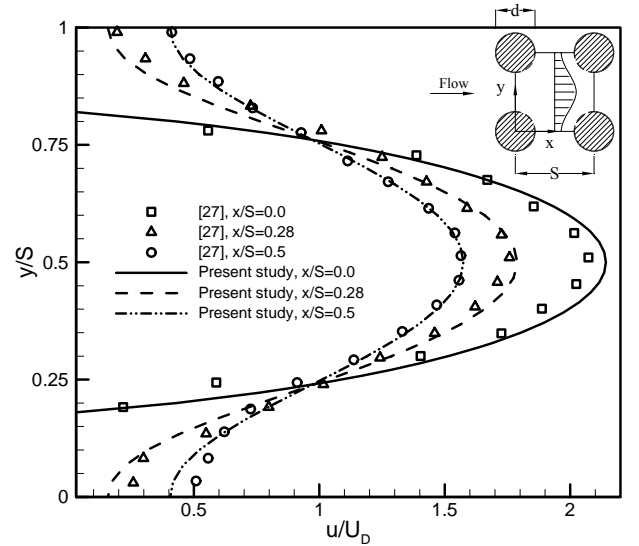


Figure 7: Comparison of the present numerical and the experimental data for the velocity profiles in normal flow and $\epsilon = 0.9$.

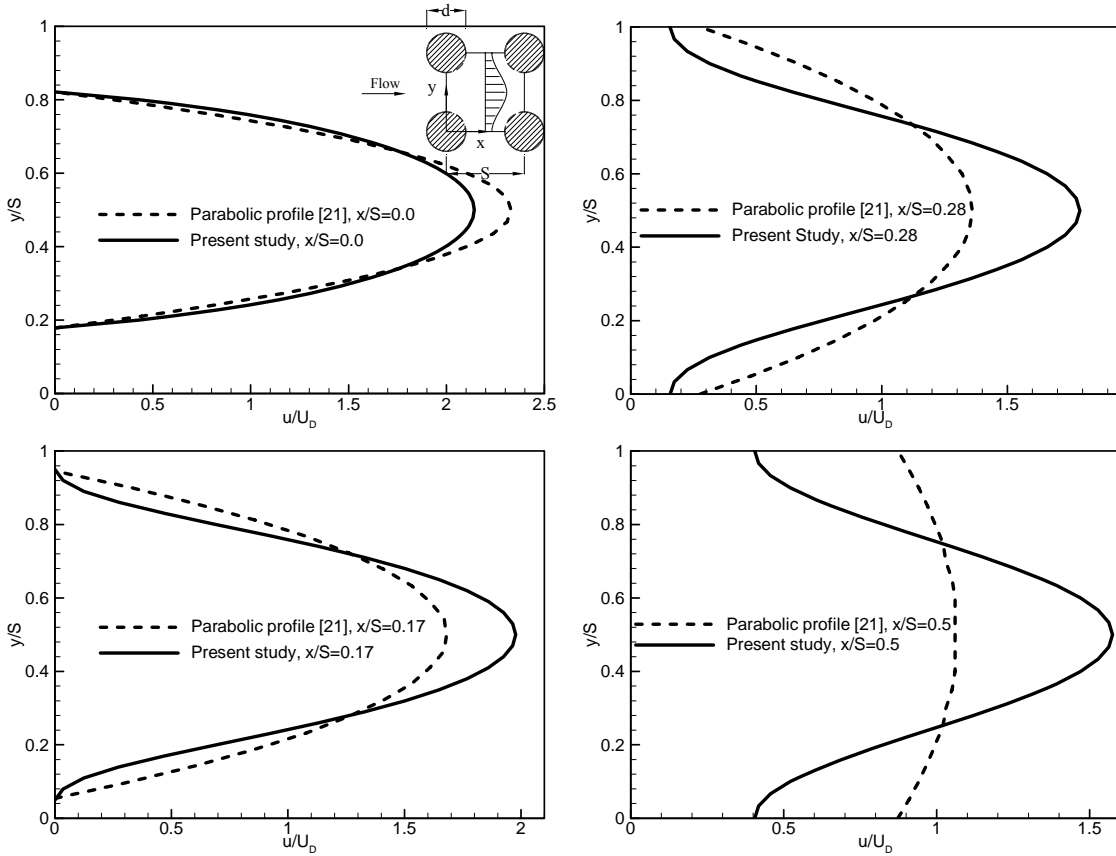


Figure 8: Comparison of the present numerical and parabolic velocity profiles in normal flow, $\varepsilon = 0.9$.

The model proposed by Drummond and Tahir [13] estimates numerical results within 10% accuracy, which is resulted from their considered velocity profile. However, the model of Drummond and Tahir [13] were not accurate for normal flows and its accuracy is restricted to parallel permeability.

Table 2: The numerical results of parallel permeability and related parameters, $d = 1$ (cm) and $u_{inlet} = 0.05$ (m/s)

ε	S (cm)	L (cm)	U_D (m/s)	ΔP (Pa)	K^*
0.215	1	5	$\frac{0.0107}{5}$	404.3	0.0013
0.35	1.10	5	0.0175	228.7	0.0038
0.45	1.19	2	0.0225	56.8	0.0079
0.55	1.32	5	0.0275	77.6	0.0177
0.65	1.50	5	0.0325	43	0.0378
0.8	1.98	5	0.04	12	0.1667
0.9	2.80	2	0.045	1.4	0.6429

After reviewing the analytical relationships for the parallel permeability of ordered fibrous beds, Drummond and Tahir [13] proposed that the suitable model of permeability should have the following form:

$$K^* = \frac{K}{d^2} = \left(2\varphi - \frac{\varphi^2}{2} - \ln\varphi - C\right) \frac{1}{16\varphi} \quad (10)$$

in which C is a constant and should be determined empirically. This equation is very sensitive to value of C . When $C = 1.5$, Eq. (10) yields the model of Happel and

Brenner [9]. Although Drummond and Tahir [13] proposed $C = 1.476$, our analysis shows that if $C = 1.485$, Eq. (10) can accurately fit the numerical results over the entire range of porosity. Equation (10) is plotted for different values of C and compared with the numerical and the experimental data in Fig. 11.

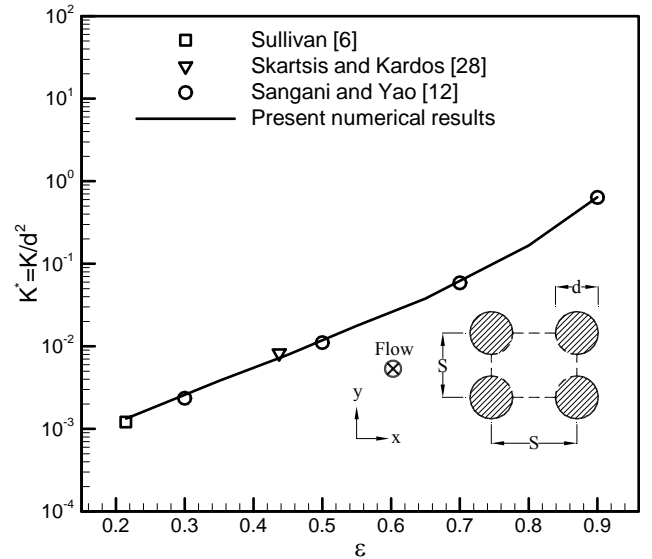


Figure 9: Comparison between the present numerical results, experimental data, and the numerical results of [12].

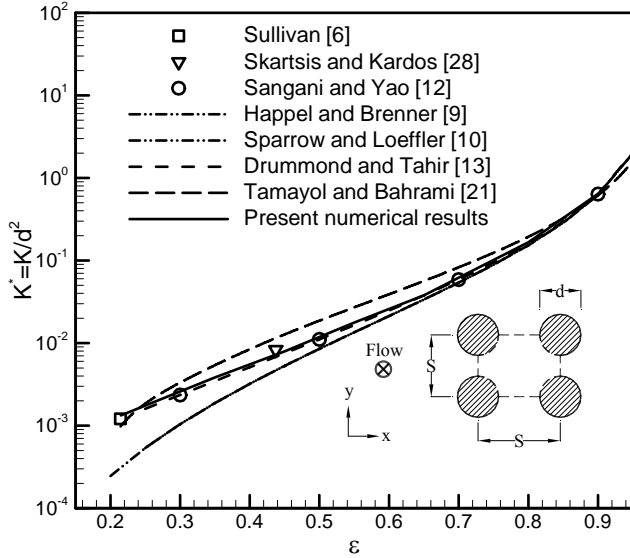


Figure 10: Comparison between the present numerical results, experimental data, and several analytical models.

The accuracy of the final results obtained from the integral technique is dependent on the assumed velocity profiles. Hence, once the assumed velocity profiles revises, the resultant relationship becomes more accurate. Tamayol and Bahrami [21] considered the following parabolic velocity profiles:

$$w = \frac{1}{2\mu} \frac{dP}{dz} (\delta^2 - y^2), \quad 0 \leq x \leq \frac{d}{2}$$

$$w = \frac{1}{2\mu} \frac{dP}{dz} \left[\left(\frac{S}{2}\right)^2 + \left(\frac{S-d}{2}\right)^2 - \left(x - \frac{S}{2}\right)^2 - y^2 \right], \quad \left(\frac{d}{2}\right) \leq x \leq \frac{S}{2} \quad (11)$$

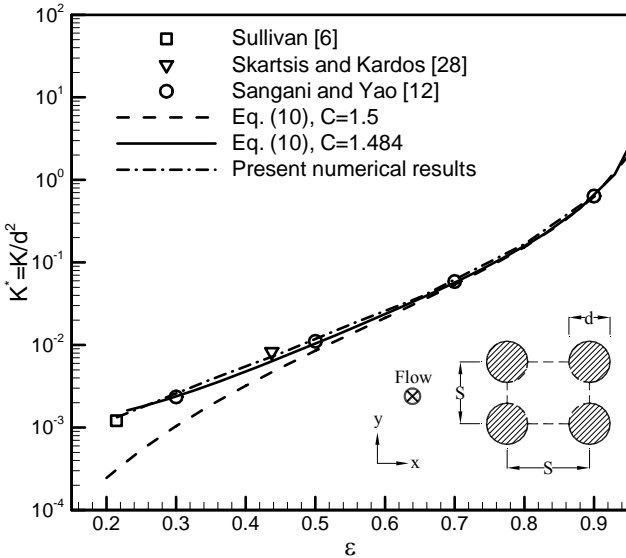


Figure 11: comparison of Eq. (7) for different values of C with existing data.

To plot this equation one has to know the pressure gradient beforehand; this can be found directly from the proposed

relationship [21] and employing Darcy's law:

$$K^* = \frac{K}{d^2} = \left\{ \frac{\pi}{24\varphi} + \left[\left(\sqrt{\frac{\pi}{4\varphi}} - 1 \right)^3 + 2 \right] \sqrt{\frac{\varphi}{9\pi}} - \left(\frac{\pi}{8} + \frac{\varphi}{8} \right) \frac{1-\varphi}{2} \right\} \quad (12)$$

where φ is the solid volume fraction, $\varphi = 1 - \varepsilon$, of the porous medium. In Figs. 12 and 13 the computed velocity profiles are compared with parabolic distribution [21] for $\varepsilon = 0.65$ and $\varepsilon = 0.9$, respectively. As can be seen, the form of the profiles is different especially in low porosities. However, as the porosity increases the non-uniformity in the parabolic distribution decreases and it becomes similar to the numerical profiles. The considerable difference between these velocity profiles reveals the reason of the difference between estimated values of permeability from two methods. The important point about the integral technique is that despite of unrealistic velocity assumption it predicts the permeability accurately. If the assumed profiles are verified, this method is capable of capturing the data accurately over the entire range of porosity.

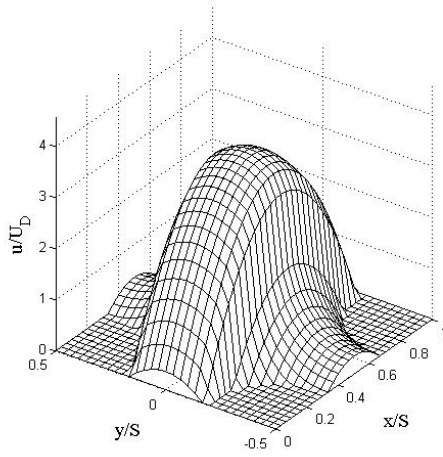
SUMMARY AND CONCLUSIONS

Parallel and normal creeping flows through 1D fibrous media are studied numerically. The fibrous material is represented by a unit cell which is assumed to be repeated throughout the media. The results are used to find the pressure drop and the permeability of the solid matrix. The numerical values of permeability of square arrangement are compared with experimental and numerical data available in the open literature and show reasonable agreement. This comparison shows that the model developed by Tamayol and Bahrami [21] captures the trends of the numerical and the experimental data for normal and parallel cases. However, differences are considerable especially for parallel flow permeability. Therefore, a compact and easy-to-use correlation is proposed for the parallel permeability over the entire range of porosity.

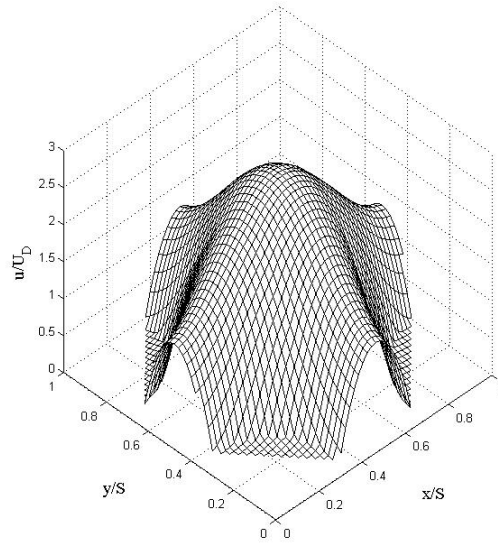
The numerical velocity profiles are also compared with experimental data and the simple parabolic distribution used along with integral method [21] and it reveals that the velocity distribution is different from the parabolic distribution. Although the assumption of the parabolic velocity is not realistic, the resulting model could capture the trends of data. However, it is expected that using a more realistic velocity profile and the integral technique, will result in a more accurate model.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada, (NSERC).

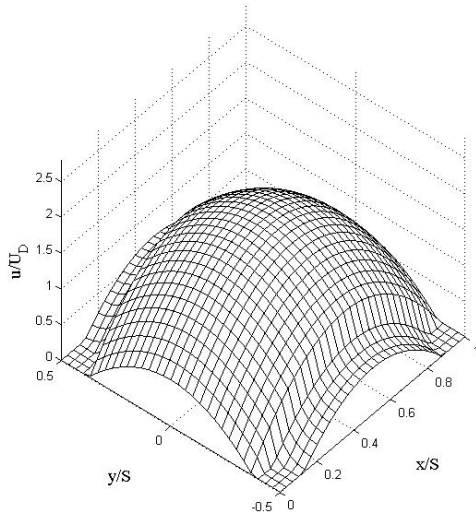


(a)

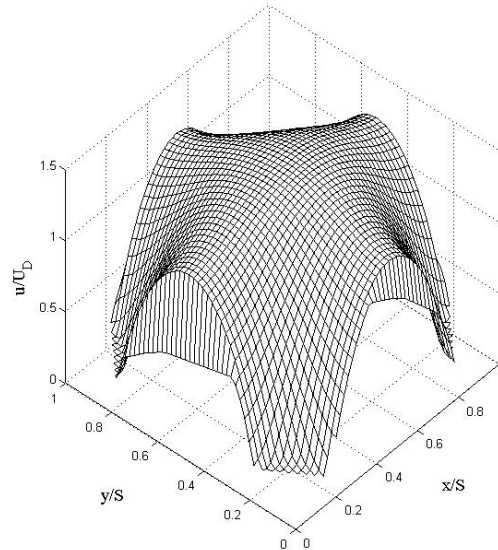


(b)

Figure 12: Comparison of non-dimensional velocity profiles: a) present numerical and b) parabolic profiles [21], parallel flow with $\varepsilon = 0.65$.



(a)



(b)

Figure 13: Comparison of non-dimensional velocity profiles: a) present numerical and b) parabolic profiles [21], parallel flow with $\varepsilon = 0.9$.

REFERENCES

1. D. S. Clauge and R. J. Philips, A Numerical Calculation of the Hydraulic Permeability of Three-Dimensional Disordered Fibrous Media, *Physics of Fluids*, Vol. 9 (6), pp. 1562-1572, 1997.
2. M. M. Tomadakis and T. Robertson, Viscous Permeability of Random Fiber Structures: Comparison of Electrical and Diffusion Estimates with Experimental and Analytical Results, *Journal of Composite Materials*, Vol. 39, pp. 163-188, 2005.
3. J. T. Gostick, M. W. Fowler, M. D. Pritzker, M. A. Ioannidis, and L. M. Behra, In-Plane and Through-Plane Gas Permeability of Carbon Fiber Electrode Backing Layers, *Journal of Power Sources*, Vol. 162, pp. 228-238, 2006.
4. M. Kaviani, Principles of Heat Transfer in Porous Media, *Springer-Verlag*, NY, 1992.
5. P. C. Carman, The Determination of Specific Surface of Powders. I, *Society Chemical Industry*, Vol. 57, pp. 225-234, 1938.
6. R. R. Sullivan, Specific Surface Measurements on Compact Bundles of Parallel Fibers, *Journal of Applied Physics*, Vol. 13, pp. 725-730, 1942.
7. S. Kuwabara, The Forces Experienced by Randomly Distributed Parallel Circular Cylinders or Spheres in a Viscous Flow at Small Reynolds Numbers, *Journal of Physical Society of Japan*, Vol. 14, pp. 527-532, 1959.
8. H. Hasimoto, On the Periodic Fundamental Solutions of the Stokes Equations and Their Application to Viscous Flow Past a Cubic Array of Spheres, *Journal of Fluid Mech.*, Vol. 5, pp. 317-328, 1959.
9. J. Happel and H. Brenner, Low Reynolds Number

- Hydrodynamics, Noordhoff, 1973.
10. E. M. Sparrow and A. L. Loeffler, Longitudinal Laminar Flow Between Cylinders Arranged in Regular Array, *AIChE Journal*, Vol. 5, pp. 325-330, 1959.
 11. A. S. Sangani and A. Acrivos, Slow Flow Past Periodic Arrays of Cylinders with Application to Heat Transfer, *International Journal of Multiphase Flow*, Vol. 8, pp.193–206, 1982.
 12. A. S. Sangani and C. Yao, Transport Processes in Random Arrays of Cylinders:II-Viscous Flow, *Physics of Fluids*, Vol. 31 (9), pp. 2435-2444, 1988.
 13. J. E. Drummond, and M. I. Tahir, Laminar Viscous Flow through Regular Arrays of Parallel Solid Cylinders, *International Journal of Multiphase Flow*, Vol. 10, 515-540, 1984.
 14. M. Sahraoui and M. Kaviani, Slip and No-Slip Boundary Condition at Interface of Porous, Plain Media, *International Journal of Heat Mass Transfer*, Vol. 37, pp. 1029–1044, 1994.
 15. J. Van der Westhuizen, and J. P. Du Plessis, An Attempt to Quantify Fiber Bed Permeability Utilizing the Phase Average Navier-Stokes Equation, *Composites*, Vol. 27A, pp. 263–269, 1996.
 16. M. Hellou, J. Martinez, and M. El Yazidi, Stokes Flow through Microstructural Model of Fibrous Media, *Mechanics Research Communications*, Vol. 31, pp 97-103, 2004.
 17. M. P. Sobera and C. R. Kleijn, Hydraulic Permeability of Ordered and Disordered Single-Layer Arrays of Cylinders, *Physical Review*, Vol. 74, 036302-1-10, 2006.
 18. G.W. Jackson and D.F. James, The Permeability of Fibrous Porous Media, *Canadian Journal of Chemical Eng.*, Vol. 64, pp. 364–374, 1986.
 19. M.M. Tomadakis and S.V. Sotirchos, Transport Properties of Random Arrays of Freely Overlapping Cylinders with Various Orientation Distributions, *Journal of Chemical Physics*, Vol. 98, pp. 616–626, 1993.
 20. M. Avellaneda and S. Torquato, Rigorous Link between Fluid Permeability, Electrical Conductivity, and Relaxation Times for Transport in Porous Media,” *Physics of Fluids*, Vol. 11, pp. 2529-2540, 1991.
 21. A. Tamayol and M. Bahrami, Analytical Determination of Viscous Permeability, *ASME FED2008, Jacksonville*, 2008.
 22. C.P. Kyan, D.T. Wasan and R.C. Kinter, Flow of Single-Phase Fluids through Fibrous Beds, *Industrial Engineering and Chemical Fundamentals*, Vol. 9, pp. 596-603, 1970.
 23. O.P. Bergelin, G.A. Brown, H.L. Hull, and F.W. Sullivan, Heat Transfer and Fluid Friction during Viscous Flow Across Banks of Tubes: III – A Study of Tube Spacing and Tube Size, *ASME Transactions*, Vol. 72, pp. 881–888, 1950.
 24. A. A. Kirsch and N.A. Fuchs, Studies on Fibrous Aerosol Filters—II Pressure Drops in Systems of Parallel Cylinders, *Annals of Occupational Hygiene*, Vol. 10, pp. 23–30, 1967.
 25. T. A. K. Sadiq, S. G. Advani, and R. S. Parnas, Experimental Investigation of Transverse Flow through Aligned Cylinders, *International Journal of Multiphase Flow*, Vol. 21 (5), pp. 755–774, 1995.
 26. B. Khomami and L. D. Moreno, Stability of Viscoelastic Flow around Periodic Arrays of Cylinders, *Rheologica. Acta.*, Vol. 36 (4), pp. 367–383, 1997.
 27. W. H. Zhong, I. G. Currie, and D. F., James, Creeping Flow through a Model Fibrous Porous Medium, *Experiments in Fluids*, Vol. 40, pp. 119-126, 2006.
 28. L. Skartsis and J.L. Kardos, The Newtonian Permeability and Consolidation of Oriented Carbon Fiber Beds, *Proceedings of American Society of Composites Technical Conference*, Vol. 5, 548–556.
 29. B. T. Astrom, R. B. Pipes, and S. G. Advani, On Flow through Aligned Fiber Beds and Its Application to Composite Processing, *Journal of Composite Materials*, Vol. 26 (9), pp. 1351-1373, 1992.
 30. Fluent 6.3 User’s Guide., (fluent Inc., Lebanon, USA) , 2007.
 31. M. Bahrami, M. M. Yovanovich, and J. R. Culham, Effective Thermal Conductivity of Rough Spherical Packed Beds, *International Journal of Heat and Mass Transfer*, Vol. 49, pp. 3691-3701, 2006.